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(M^X/E_K/C:C/FIFO) Queue with Two-Class Arrivals, State dependent service, Multi-servers and Customers Impatience

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Abstract

This work covers two arrival types with differing arrival rates, client impatience, state-dependent service and a finite Markovian queueing system with numerous servers. Arrivals and departures are assumed to have Poisson and exponential distributions respectively and are made on a first in first out. In every industrial process, bulk arrivals are taken into consideration and service may be given phase-wise. A few performance measures and transient state probabilities have been calculated. We have displayed the results of the sensitivity analysis and observed the impact of various factors on the system's constants.

Key words: Two-class customers, Bulk Arrivals, Phase wise service, Balking and multi-server facility

I Introduction

Queuing theory, a branch of operations research, is used to identify the most effective ways to provide services in light of the constraints. Queues help organizations to deliver services in a methodical manner. The main goal of the analysis is to fix a mathematical model using the given arrival and service rates. Due to their numerous applications, queuing models have attracted a lot of attention recently.

Haight was the first to study the concept of customer impatience [6]. He came up with a balking model for M/M/1 queues, in which an arrival wouldn't join at the very end of the line. Reneging and balking—the refusal to immediately enter the line upon arrival—are two of the queuing issues that Ancker and Gafarian [2] discussed. Since then, many academics have made significant contributions in this field [1, 3, 13, 15].

Many studies have focused on methods to reduce client irritation. The concept of several servers is one of them. Y. Levi et al. have studied an M/M/s queue with vacation and have developed performance measurements [21]. Davis et al. identified key inputs for a multi-server queueing paradigm in a priority queue [14]. The literature in this field is extensive [7,8,9,10,11]. Depending on the service they require, customers enter the line in various ways. The same kind of service may also have different arrival rates in real life. Numerous studies have been conducted in this area. An M/M/m queue with two customer classes and numerous vacations was examined by Mingzhou Xie et al. [12]. Studies [4,5,12] are crucial in this subject.

Batch arrival queuing systems are widely used in many real-world applications, including computer networks, manufacturing/production systems, and communication. A bulk arrival queueing model is frequently used by researchers to examine this system since it provides a strong instrument for evaluating system performance. In a production/manufacturing system, for instance, an operation won't begin until a specific quantity of raw materials has accumulated over a period of inactivity. Serjio et al. [16] introduced a new method for planning the picking procedure of orders in line. The statistical tests validated the statistical significance of the data. Many other scholars have also made important contributions [17,18].

Phase-wise service is widely required in many service as well as production and manufacturing sectors. Lot of literature exists in this area. Researchers [19,20] have made their contribution for this.

The transient analysis of an $M^X/E^K/C$ queue with two arrival types and state-dependent service rates has been described in this study. The following is how the work is structured: The model is explained in Section 2, the transient state model and associated probabilities are covered in Section 3, and several computational constants of the system are shown using numerical findings and sensitivity analysis in Section 4. Final conclusions are presented in Section 5.

II About the Model

We studied a finite Markovian Queueing system having multi -servers with heterogenous arrivals who joins bulk in nature and requires phase-wise service , state dependent service and Customer Impatience in Transient mode in the following scenario:

1. The capacity of the system as well as the number of service channels are considered to be equal in number as c.
2. The two types of mean arrival rates are λ_1 and λ_2 and a probability density function exists for independent and identically distributed random batches of customers, with each batch size X. $\{a_n: a_n = P(X=n), n\geq1\}$.
3. The two types of mean service rate of servers of Type-I customers are μ_1 and μ_2.
4. The balking parameter is b.

5. The service will be provided in k-phases.
6. $\pi_{x,y}^{(t)}$ denotes the probability that there are "x" customers of Type-I and "y" customers of Type-II at time point "t".

The following differential equations are formed to compute various probabilities:

$$\frac{d\pi_{0,0}^{(t)}}{dt} = -(\lambda_1+\lambda_2)b\pi_{0,0}^{(t)} + \mu_1\pi_{1,0}^{(t)} + \mu_2\pi_{0,1}^{(t)} \quad (1)$$

$$\frac{d\pi_{x,0}^{(t)}}{dt} = -((\lambda_1+\lambda_2)b + x\mu_1)\,\pi_{x,0}^{(t)} + \lambda_1 b\sum_{i=1}^{x} a_i\pi_{x-i,0}^{(t)} + (x+k)\mu_1\pi_{x+k,0}^{(t)} + \mu_2\pi_{x,k}^{(t)};\ k \le x \le (c-1)k \quad (2)$$

$$\frac{d\pi_{0,y}^{(t)}}{dt} = -((\lambda_1+\lambda_2)b + y\mu_2)\,\pi_{0,y}^{(t)} + \lambda_2 b\sum_{i=1}^{y} a_i\,\pi_{0,y-i}^{(t)} + (y+k)\,\mu_2\pi_{0,y+k}^{(t)} + \mu_1\pi_{k,y}^{(t)};\ k \le y \le (c-1)k \quad (3)$$

$$\frac{d\pi_{x,y}^{(t)}}{dt} = -((\lambda_1+\lambda_2)b + x\mu_1 + y\mu_2)\,\pi_{x,y}^{(t)} + \lambda_1 b\sum_{i=k}^{x} a_i\pi_{x-ik,0}^{(t)} + \lambda_2 b\sum_{i=k}^{y} a_i\,\pi_{0,y-ik}^{(t)} + (x+k)\,\mu_1\pi_{x+k,y}^{(t)} + (y+k)\,\mu_2\pi_{x,y+k}^{(t)};\ x \text{ and } y \ne 0 \text{ and } x+y \le (c-1)\text{k} \quad (4)$$

$$\frac{d\pi_{ck,0}^{(t)}}{dt} = -(ck\mu_1)\,\pi_{ck,0}^{(t)} + \lambda_1 b\sum_{i=k}^{ck} a_i\,\pi_{ck-i,0}^{(t)} \quad (5)$$

$$\frac{d\pi_{0,ck}^{(t)}}{dt} = -(ck\mu_2)\,\pi_{0,ck}^{(t)} + \lambda_2 b\sum_{i=k}^{ck} a_i\,\pi_{0,ck-i}^{(t)} \quad (6)$$

$$\frac{d\pi_{x,y}^{(t)}}{dt} = -(x\mu_1 + y\mu_2)\pi_{x,y}^{(t)} + \lambda_1 b\sum_{i=k}^{x} a_i\,\pi_{x-i,y}^{(t)} + \lambda_2 b\sum_{i=k}^{y} a_i\,\pi_{x,y-i}^{(t)}\quad x \text{ and } y \ne 0 \text{ and } x+y = ck \quad (7)$$

III Performance Measures

The following Queueing measurements are computed:

1. Expected length of system ($L_S^{(t)}$)
2. Mean waiting time ($W_S^{(t)}$)

IV Numerical Results

Computations are done for Mean length and waiting time and also for sensitivity analysis by using MATLAB by verifying the traffic intensity condition. They are presented in Tables 1-6.

The model parameters are considered as follows:

$$c = 3,\ \lambda_1 = .04, \lambda_2 = .05, \mu_1 = .08, \mu_2 = .14, b = 0.004, k = 2$$

The time instances are taken as follows:

$$t_1 = 0.5, t_2 = 1.0, t_3 = 1.5, t_4 = 2.0$$

Table 1: Effect of λ_1					
Parameter (λ_1)	t	t_1	t_2	t_3	t_4

0.04	$L_S^{(t)}$	0.0014055	0.0016211	0.0018428	0.0168427
	$W_S^{(t)}$	0.0011241	0.0011673	0.0016498	0.0021653
0.042	$L_S^{(t)}$	0.0016122	0.0017462	0.0101842	0.0194000
	$W_S^{(t)}$	0.0012412	0.0014021	0.0140164	0.0162021
0.044	$L_S^{(t)}$	0.001708	0.0017746	0.011184	0.0204194
	$W_S^{(t)}$	0.0014622	0.002140	0.0261400	0.0294264
0.046	$L_S^{(t)}$	0.0018006	0.0082016	0.0134162	0.0302150
	$W_S^{(t)}$	0.0167183	0.0028128	0.03418439	0.0462355

Inference: From the above table, it is observed that the expected queue length as well as mean waiting times are raising with respect to a raise in Type-I arrival rate λ_1.

Table 2: Effect of λ_2

Parameter (λ_2)	t	t_1	t_2	t_3	t_4
0.05	$L_S^{(t)}$	0.0014055	0.0016211	0.0018428	0.0168427
	$W_S^{(t)}$	0.0011241	0.0011673	0.0016498	0.0021653
0.052	$L_S^{(t)}$	0.0014058	0.0017346	0.0019864	0.0196601
	$W_S^{(t)}$	0.0012004	0.0014614	0.010940	0.0119640
0.054	$L_S^{(t)}$	0.0016042	0.0019273	0.0020410	0.0224873
	$W_S^{(t)}$	0.0012404	0.0018916	0.0214201	0.0219640
0.056	$L_S^{(t)}$	0.0017404	0.0020042	0.0021046	0.0226344
	$W_S^{(t)}$	0.0012861	0.0019944	0.0246542	0.0298246

Inference: From the above table, it is observed that the expected queue length as well as mean waiting times are raising with respect to a raise in Type-II arrival rate λ_2.

Table 3: Effect of μ_1

Parameter (μ_1)	t	t_1	t_2	t_3	t_4
0.08	$L_S^{(t)}$	0.0014055	0.0016211	0.0018428	0.0168427
	$W_S^{(t)}$	0.0011241	0.0011673	0.0016498	0.0021653
0.12	$L_S^{(t)}$	0.0014002	0.0016146	0.0016402	0.0164680
	$W_S^{(t)}$	0.0010982	0.0011073	0.0013986	0.0020846
0.16	$L_S^{(t)}$	0.0013896	0.0016098	0.0016396	0.0164244
	$W_S^{(t)}$	0.0010444	0.0010962	0.0011460	0.0020666
0.20	$L_S^{(t)}$	0.0013890	0.0016086	0.0016214	0.0163012
	$W_S^{(t)}$	0.0010440	0.0010660	0.0011324	0.0019846

Inference: From the above table, it is observed that the expected queue length as well as mean waiting times are decreasing with respect to a decline in Type-I service rate μ_1.

Table 4: Effect of μ_2

Parameter (μ_2)	t	t_1	t_2	t_3	t_4
0.14	$L_S^{(t)}$	0.0014055	0.0016211	0.0018428	0.0168427
	$W_S^{(t)}$	0.0011241	0.0011673	0.0016498	0.0021653
0.18	$L_S^{(t)}$	0.0014030	0.0015000	0.0016428	0.0167208
	$W_S^{(t)}$	0.0011124	0.0011264	0.0014449	0.0018262
0.22	$L_S^{(t)}$	0.0014014	0.0014645	0.0015640	0.0016646
	$W_S^{(t)}$	0.0011116	0.0011242	0.0014362	0.0016400
0.26	$L_S^{(t)}$	0.0014009	0.0013568	0.0014442	0.0015876
	$W_S^{(t)}$	0.0011108	0.0011114	0.0012684	0.0014221

Inference: From the above table, it is observed that the expected queue length as well as mean waiting times are decreasing with respect to a decline in Type-I service rate μ_2.

Table 5 Effect of b

Parameter (b)	t	t_1	t_2	t_3	t_4
0.004	$L_S^{(t)}$	0.0014055	0.0016211	0.0018428	0.0168427
	$W_S^{(t)}$	0.0011241	0.0011673	0.0016498	0.0021653
0.0041	$L_S^{(t)}$	0.0016122	0.0017462	0.0101842	0.0194000
	$W_S^{(t)}$	0.0012412	0.0014021	0.0140164	0.0162021
0.0042	$L_S^{(t)}$	0.0016405	0.0018467	0.0112401	0.0200061
	$W_S^{(t)}$	0.0012600	0.0014614	0.0165409	0.0214610
0.0043	$L_S^{(t)}$	0.001710	0.0019420	0.0204626	0.0224613
	$W_S^{(t)}$	0.0016842	0.0018426	0.0201249	0.0246890

Inference: From the above table, it is observed that the expected queue length as well as mean waiting times are decreasing with respect to increase in balking probability.

Table 6 Effect of k

Parameter (k)	t	t_1	t_2	t_3	t_4
2	$L_S^{(t)}$	0.0014055	0.0016211	0.0018428	0.0168427
	$W_S^{(t)}$	0.0011241	0.0011673	0.0016498	0.0021653
3	$L_S^{(t)}$	0.0014612	0.0018227	0.0101666	0.0192424
	$W_S^{(t)}$	0.0012004	0.0012428	0.0146064	0.0162021
4	$L_S^{(t)}$	0.0016402	0.0018456	0.0112398	0.0200422
	$W_S^{(t)}$	0.0012642	0.0015681	0.0169006	0.0209400
	$L_S^{(t)}$	0.0017140	0.0020120	0.0204222	0.0212403

5	$W_S^{(t)}$	0.0012684	0.0100066	0.0194264	0.0211424

V Conclusion

The objective of this work is detailing a queueing system with two arrival types and state-dependent service with client impatience. This work can be extended by considering a cost function and to deduce the number of servers required to optimize such total cost.

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