A CHARACTERIZATION OF LIAR DOMINATION ON INTUITIONISTIC FUZZY GRAPHS

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Abstract
In some networks including computer network, communication network, transport network, social network etc. Consider there is some intruder such as thief, fire in a place or fire in ventilation duct or possible processor fault in a computer network. A set S is said to be a liar dominating set if it can identify the location node x of an intruder when any one of the nodes which is closed neighborhood of x lie or wrongly identify the intruder's location.
In this paper, we study some characterization of liar dominating set for intuitionistic fuzzy graphs and also discuss some theorems and results with suitable examples.

Introduction

Assume that each vertex of a graph G is the probable location for an "intruder", for instance, a thief or a saboteur, a fire in a facility or some imaginable processor fault in a computer network. A device at a vertex u could be expected to sense the intruder at any vertex in its closed neighbourhood $N[u]$ and to recognize at which vertex in $N[u]$ the intruder is positioned. Liar's dominating sets can identify an intruder's position even if the other device in the neighbourhood of the intruder vertex fails, that is, if device in the neighbourhood of the intruder vertex misidentifies the presence of other vertex in its closed neighbourhood as the intruder location. P.J. Slater [1] introduced liar's dominating sets in graph theory. P.J. Slater [2] introduced Locating sets and it was carried forward by F. Harary and R.A. Melter[3] where they termed it as metric bases. D. Manuel Paul [4] premeditated on both locating-domination and liar domination in circulant networks to narrate the characterization of locating-dominating set and liar dominating set of circulant networks and sharp lower and upper bounds. .[6] A.Nagoorgani studied Fuzzy Independent dominating sets.[5][7] A.Nagoorgani studied double domination on intuitionistic fuzzy graphs.

Basic Definitions

Definition 1 : [7] An edge uv is said to be strong if $\alpha_2^{\infty}(u,v) = \alpha_2(u,v)$ and $\beta_2^{\infty}(u,v) = \beta_2(u,v)$, where $\alpha_2^{\infty}(u,v)$ is maximum weight of weakest arc and $\beta_2^{\infty}(u,v)$ is minimum weight of weakest arc.
That is,
$\alpha_2^{\infty}(u_i,u_j)= \sup\{\alpha_2^{k}(u_i,u_j)/k=1,2,\dots n\}$
$\beta_2^{\infty}(u_i,u_j)= \inf\{\alpha_2^{k}(u_i,u_j)/k=1,2,\dots n\}$
Definition 2 : [7] Open neighbourhood of a vertex u is defined as, $N(u) = \{v \in V(G)\backslash \mu^{\infty}(u,v) = \mu(u,v)\}$
Definition 3 : [7] Closed neighbourhood of a vertex $N[u]$ is defined as, $N[u] = \{u\} \cup \{v \in V(G) : \mu^{\infty}(u,v) = \mu(u,v)\}$
Definition 4 : [7]The vertex u is said to be dominated by the vertex v if $u \in N[v]$, where, $N[v] = \{v\} \cup \{u \in V, (u,v) \textit{is strong edge}\}$.
Definition 5 : [7]Let $G =< \sigma,\mu >$ be a fuzzy graph on V . A set $A \subseteq V$ is called a liar dominating or domination set if it satisfies the following conditions.
1. Each vertex $u \in V(G)$ is dominated by at least two vertices in A.
2. Every pair of vertices $u,v \in V(G)$ is dominated by at least three vertices in A.

Definition 6 : [7] An intuitionistic fuzzy graph (IFG) is of the form $G = (V,E)$ where

1. $V = \{u_1, u_2, u_3, ..., u_n\}$ *such that* $\alpha_1 : V \to [0,1]$ *and* $\beta_1 : V \to [0,1]$ denote the degree of membership and non membership of the vertex $u_i \in V$ respectively and $0 \le \alpha_1(u_i) + \beta_1(u_i) \le 1$, for every $u_i \in V$ (1 = 1,2,..*n*).
2. $E \subset V \times V$ *where* $\alpha_1 : V \times V \to [0,1]$ *and* $\beta_1 : V \times V \to [0,1]$ such that

$$\alpha_{2ij} = \alpha_2(u_i, u_j) \le \min(\alpha_1(u_i), \alpha_1(u_j))$$
$$\beta_{2ij} = \beta_2(u_i, u_j) \le \max(\beta_1(u_i), \beta_1(u_j))$$
$$\text{And } 0 \le \alpha_2(u_i) + \beta_2(u_i) \le 1 \text{ for all } (u_i, u_j) \in E$$

Definition 7 : [7] An intuitionistic fuzzy graph G = (V,E) is called strong if

$$\alpha_{2ij} = \alpha_2(u_i, u_j) = \min(\alpha_1(u_i), \alpha_1(u_j))$$
$$\beta_{2ij} = \beta_2(u_i, u_j) = \max(\beta_1(u_i), \beta_1(u_j))$$

for all $(u_i, u_j) \in E$.

Definition 8 : [7] An intuitionistic fuzzy graph G = (V,E) is called complete if

$$\alpha_{2ij} = \alpha_2(u_i, u_j) = \min(\alpha_1(u_i), \alpha_1(u_j))$$
$$\beta_{2ij} = \beta_2(u_i, u_j) = \max(\beta_1(u_i), \beta_1(u_j))$$

for all $u_i, u_j \in V$.

Definition 9:[7] Let G =(V,E) be an IFG. Then the cardinality of G is defined to be

$$|G| = \left| \sum_{u_i \in V} \frac{1+\alpha_1(u_i)-\beta_1(u_i)}{2} + \sum_{u_i, u_j \in E} \frac{1+\alpha_2(u_i,u_J)-\beta_2(u_i,u_J)}{2} \right|$$

Definition 10:[7]

Let G =(V,E) be an IFG. Then the vertex cardinality of G is defined to be

$$|V| = \sum_{u_i \in V} \frac{1+\alpha_1(u_i)-\beta_1(u_i)}{2}, \text{ for all } u_i \in V$$

Definition 11:[7]

Let G =(V,E) be an IFG. Then the edge cardinality of G is defined to be

$$|E| = \sum_{u_i, u_j \in V} \frac{1+\alpha_2(u_i,u_J)-\beta_2(u_i,u_J)}{2}, \text{ for all } u_i, u_j \in E1$$

Definition 12 : [7] A path in an intuitionistic fuzzy graph G is a sequence of vertices and edges $u_1 e_1 u_2 e_2 ...$ such that either one of the following conditions is satisfied.

1. $\alpha_{2ij} > 0$ *and* $\beta_{2ij} = 0$ *for* some *i*, *j*.
2. $\alpha_{2ij} = 0$ *and* $\beta_{2ij} > 0$ *for* some *i*, *j*.
3. $\alpha_{2ij} > 0$ *and* $\beta_{2ij} > 0$ *for* some *i*,*j*.

Definition 13 :[7] Let *u* be a vertex in intuitionistic fuzzy graph. Then, $N[u] = \{u\} \cup \{v : v \in V$ and (u,v) is strong edge in IFG $\}$ is called closed neighbourhood of the vertex *u* in *G*.

Definition 14 : [6]A vertex *u* is said to be a isolated vertex if $N(u) = \emptyset$. An isolated vertex *u* is dominated by itself.

Definition 15 : [6]We say that *u* dominates *v* if $v \in N[u]$. This implies that *u* dominates itself. Obviously, domination satisfies symmetric relation on *V* . That is, *u* dominates *v* iff *v* dominates *u*,$\forall u,v \in V$.

If no edge is strong edge in IFG, then we can not form liar dominating set for intuitionistic fuzzy graphs, since every node is single dominant node in this case.

Liar Domination for Intuitionistic Fuzzy Graphs:

Definition : A set $I \subseteq V$ in a IFG is called a liar dominating set if *I* satisfies the following two conditions:

1. every $u \in v$ is dominated by minimum two nodes in IFG.

2. every pair $u,v \in V$, is dominated by minimum three nodes in IFG.

Definition : A liar dominating set I in IFG is called a minimum liar dominating set, if there is no liar dominating set I^0 such that $|I^0| < |I|$.

Definition : The minimum fuzzy cardinality of liar dominating set I in IFG is called liar dominating number and is denoted by $\omega(IFG)$.

In Figure 1, $\omega(IFG) = \{v_1,v_2,v_4\}$ forms liar dominating set.

Definition : A liar dominating set I^0 in IFG, is called a minimal liar dominating set if no proper subset of I^0 is a liar dominating set.

Definition : A liar dominating set I in IFG is called a maximum liar dominating set if there is no liar dominating set I^0 such that $|I^0| > |I|$.

In Figure 2, $\{a,b,c,d,e,f\}$ is maximum liar dominating set.

Theorem 1 : A liar dominating set I in IFG $G =< V,E >$ is a minimal liar dominating set if for each $p_m \in I, m = 1,2,...,i,...,j,...,n$, then one of the following conditions hold.

(i) p_i (for some i) is a strong neighbour to only one vertex in I;

(ii) There are nodes $v \in V_I$ such that $N(v) \cap I = \{p_i,p_j\}, p_i,p_j \in I$.

Proof 1 : Let I be a minimal liar dominating set in IFG. Let $p_i \in I$. Then $I - \{p_i\}$ is not a liar dominating set of G. This implies that there is some $u \in V$ is not dominated by $I - \{p_i\}$.

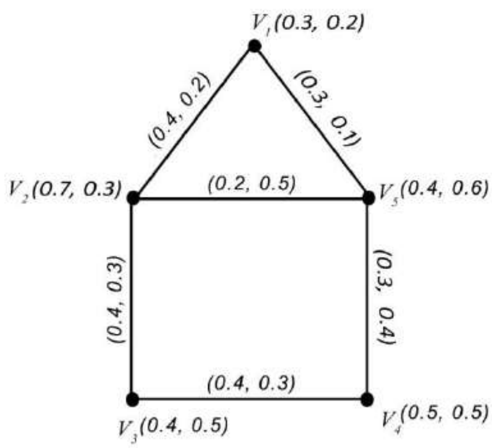

Figure 1:

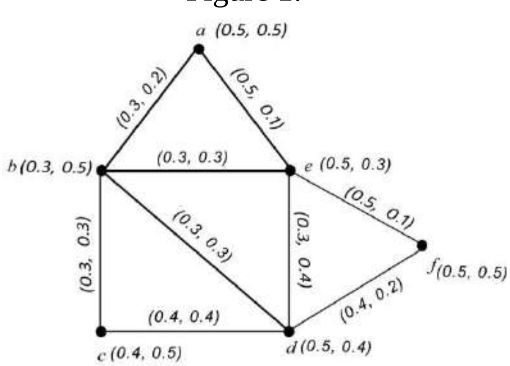

Figure 2:

If $u = p_i$, then p_i is strong neighbour to only one vertex in I. Since I is a liar domination set, if v 6= p_i, (i.e) $v \in V - I$, then there are two strong neighbours to v in I. That is, $N(v) \cap I = \{p_i,p_j\}$ where $p_i,p_j \in I$.

Conversely, Suppose the two conditions (i) & (ii) hold in IFG.
Suppose I is not a minimal liar dominating set. If v 6= $p_i \in I$, then there are two strong neighbours to v in I, then this is contradiction to (i).
Let $v \in V - I$, then each v is dominated by at least three vertices in I. Then Condition (ii) fails in this case.
Therefore I must be minimal liar dominating set in IFG.

Theorem (2)

If G is a intuitionistic fuzzy graph without pendant vertices and n≥ 3, then $\frac{0.3(n)}{0.3+\Delta(G)} \leq \omega(IFG) \leq O(G) - \sigma(u_i)$, for some i.

Proof:

In an Intuitionistic fuzzy graph, pendant vertex v is a vertex in G such that N(v) = {u}, for some u$\in V(G)$.
If G does not contain any pendant vertex, then all the vertices must be in a cycle. This implies that one can find a vertex namely u_i, for some i is dominated by at least two vertices in I. Obviously, u satisfies the second axiom of liar dominating set if D = V\\u_i.
Thus,

$$\omega(IFG) \leq O(G) - \sigma(u_i)$$

Each node of G can dominate itself and $\Delta(G)$ other nodes.Therfore, lower bound for dominating number could be,
$\frac{0.3(n)}{0.3+\Delta(G)} \leq \omega(IFG)$, which proves the theorem.

Theorem (3):

$\omega(IFG)$+ $\omega(\overline{IFG})$ <2 (O(G))

Proof:

Let IFG be an intuitionistic fuzzy graph and $\overline{IFG}$ be a complement of IFG.
From the above theorem,
$\omega(IFG) \leq O(G) - \sigma(u_i)$ (1)
But, $\overline{IFG}$ may be an intuitionistic fuzzy graph with pendant vertices.
□(□□□) ≤ □(□) (2)
From (1) and (2)
□(□□□)+ □(□□□) <2 (O(G))

Theorem (4):

(p-□(v))□(□, □) ≤ □(□□□), where (u,v) is weakest arc and G has no pendant vertices.

Proof:

If G is an intuitionistic fuzzy graph without having pendant vertices, then every vertex has 2 neighbours. Since there is no pendant vertex in G, G must contain a fuzzy cycle. Then there is at least one node in G has two strong neighbours. Therefore, V(G) \u form a liar dominating set in G. Since (u,w) is weakest arc, □(□, □) is the least membership value.
Therefore,
(p-□(v))□(□, □) ≤ □(□□□),

Conclusion

In this paper, we introduced the liar dominating set for intuitionistic fuzzy graphs and also discussed some theorems and results with suitable examples.

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